

Orbital velocity

For a stable orbit there is a balance between the centrifugal force of the circular motion and the gravitational force of the body. This implies that $F_{cent} = F_{grav}$ which leads to the following derivation:

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$
$$v^2 = \frac{GM}{r}$$
$$v_{orb} = \sqrt{\frac{GM}{r}}$$

Legend:

M: the mass of the body.

m: the mass of the object orbiting the body.

G: gravitational constant.

v: orbital velocity.

Escape velocity

The escape velocity is the velocity needed to break free of the gravitational pull of a body. It is the speed at which $E_{kin} = E_{pot,grav}$. This leads to the following derivation:

$$E_{kin} = E_{pot,grav}$$
$$\frac{mv^2}{2} = \frac{GMm}{r}$$
$$v^2 = \frac{2GM}{r}$$
$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

Legend:

M: the mass of the body.

m: the mass of the object orbiting the body.

G: gravitational constant.

v: escape velocity.